Manipulator Performance Constraints in Cartesian Admittance Control for Human-Robot Cooperation

Fotios Dimeas, Vassilis C. Moulianitis, Charalampos Papakonstantinou and Nikos Aspragathos

Abstract—This paper addresses the problem of providing feedback to the operator about the manipulator’s performance during human-robot physical interaction. A method is proposed that implements virtual constraints in Cartesian admittance control in order to prevent the operator from guiding the manipulator to low-performance configurations. The constraints are forces expressed in the Cartesian frame, which restrict the translation of the end-effector when the operator guides the robot below a certain performance threshold. These forces are calculated online by numerically approximating the gradient of the performance index with respect to the Cartesian frame attached to the end-effector. An experimental evaluation is conducted involving human-robot interaction with a 7-DOF LWR serial manipulator under Cartesian admittance control, using the kinematic manipulability index of the manipulator as the performance measure for singularity avoidance.

I. INTRODUCTION

Physical interaction between a human and a robotic manipulator is an emerging field aiming to support tasks like cooperative lifting of heavy objects or programming by demonstration. During such tasks, the human guides the robot by applying to it external forces and torques. An effective scheme to impose a desired dynamic behavior between the external forces and the motion of the robot is impedance or admittance control [1]. Efficient and safe interaction requires that the robot operates within its capabilities and behaves predictably to the interaction.

Virtual position and force constraints (virtual fixtures or active constraints) in human-robot interaction, is a widely used technique to render virtual environments in impedance or admittance controlled robots and provide assisting feedback to the operator in predefined trajectories [2]. In their vast majority, the constraints are used to guide the operator into pre-configured virtual paths or configurations, by applying attractive or repulsive feedback to improve the operator’s performance. However, the issue of providing feedback to the operator about the performance of the manipulator itself has not been addressed. A robot’s performance can be quantified by appropriate indices that have been proposed in the literature [3] such as the kinematic manipulability [4] and the end-effector’s velocity ratio MVR [5]. These measures are mainly used in the design of optimized robot structures, in optimal task placement and in null space resolution of redundant manipulators [6].

The preconditions for both efficient and safe physical interaction, raise the need to regulate the operator’s motion with respect to the performance of the manipulator. Two of the main challenges in a robot’s operation within unstructured environments are: the operation in a workspace with high performance, where the motors’ effort is minimal, and the avoidance of kinematic singularities, where the robot loses the ability to move along certain directions of the end-effector. In a singular configuration, the desired dynamic behavior of the end-effector (impedance or admittance) can no longer be achieved, deteriorating the effectiveness of human-robot cooperation. The effects of singular configurations depend on the selected controller. When the Cartesian impedance control is used, the robot can remain stuck because the operator’s forces at the end-effector are not mapped appropriately into joint torques [7]. Using the Cartesian admittance control [8], the inverse kinematic problem at a singularity presents infinite solutions, while in the neighborhood of a singularity small operational space velocities cause high joint velocities.

To prevent the robot from reaching low performance configurations during human-robot interaction, virtual position or force constraints can be used to create a repulsive field from such configuration. These constraints can either be determined by obtaining the manipulator’s performance measure manifolds, an offline process that requires significant computational work, or by online monitoring of the performance measure. In [7] it is presented, without experimental demonstration, a secondary joint impedance controller to impose a repulsive potential field calculated from the kinematic manipulability, that pushes the joints away from singular configurations. However, this approach can only be implemented with impedance controllers because the singularity avoidance algorithm outputs joint torques. Additionally, this method does not consider the end-effector’s orientation. For instance, the produced joint torques can unintentionally alter the orientation of the end-effector when a task requires a constant one.

In this paper, a novel method is proposed for implementing manipulator performance constraints during human-robot cooperation, that prevent the operator from guiding the robot towards low-performance configurations. The performance constraints are calculated online by the gradient of the performance index with respect to the Cartesian frame attached to the end-effector and do not require offline calculations of the constraints. A robust algorithm is developed to calculate...
the gradient of a performance index numerically and produce smooth virtual Cartesian forces that maintain efficient and safe human-robot cooperation, without the need of an analytical expression of the index. The proposed method can be used for singularity avoidance in human-robot interaction without prior calculation of singularity loci. An experimental evaluation is conducted involving human-robot interaction with a 7-DOF serial manipulator in tasks with low kinematic manipulability. By applying the performance constraints in a Cartesian admittance controller, the robot prevents the operator from reaching critically low values of manipulability and intuitively avoid singular configurations.

II. PERFORMANCE CONSTRAINTS IN CARTESIAN ADmittANCE CONTROL

Cooperation with a robot is usually conducted with the operator applying forces and torques to guide the end-effector. Cartesian admittance control is used to map these forces/torques (typically measured with a 6DOF force/torque sensor) to the operational space forces (Fig. 1). The performance constraints are forces/torques applied by the robot to the operator, which can be integrated to the desired admittance behavior. In the following sections, the proposed algorithm that calculates and implements the virtual constraints according to the performance measure is described.

A. Manipulator Performance Constraints

Let \( w(q) \in \mathbb{R} \) represent the performance index of an n-DOF manipulator that should remain above (or below) a certain threshold \( w_{cr} \). This index can be any from the well-known local kinematic or dynamic performance measures that depend on the configuration \( q \in \mathbb{R}^n \) of the manipulator [3], [4], [9], [10].

During the cooperation, it is possible that the operator grasps the end-effector and guides it towards a configuration with low performance value \( w(q) \). Assuming that the interaction usually takes place at the end-effector, appropriate virtual constraints can be imposed to prevent the manipulator from crossing a critically low performance value \( w_{cr} \). In this paper, the constraints are represented by the introduction of virtual forces/torques \( F_v \in \mathbb{R}^6 \) that are applied by the robot to the operator and repel the end-effector from such configurations. Assuming infinitesimal Cartesian displacements and rotations, the magnitude of \( F_v \) is determined from the distance metric \( k(w) \in \mathbb{R} \) of the current performance value \( w(q) \) from a safety performance threshold \( w_{th} \), while the direction of \( F_v \) is proportional to the gradient of the performance measure \( A(q) \propto \nabla w(q) \in \mathbb{R}^6 \) with respect to the principal axes of the Cartesian frame attached to the end-effector:

\[
F_v = k(w)A(q). \tag{1}
\]

The non-negative scalar \( k(w) \) is activated when the index crosses the threshold \( w_{th} \) and increases asymptotically while the index approaches the lowest critical value \( w_{cr} \):

\[
k(w) = \begin{cases} 
\lambda \frac{1}{w(q) - w_{cr}} & , w(q) \leq w_{th} \\
0 & , w(q) > w_{th} 
\end{cases} 
\tag{2}
\]

The positive gain \( \lambda \in \mathbb{R} \) controls the magnitude of the metric \( k(w) \), as it is shown in Fig. 2. For a low value of \( \lambda \), the constraining forces \( F_v \) are realized by the operator as a very compliant spring when \( w(q) \leq w_{th} \), that also becomes very stiff while \( w(q) \to w_{cr} \). The asymptotic increase of \( k(w) \) guarantees that the constraints are always sufficient to prevent the robot from crossing the critical value \( w_{cr} \). The values \( w_{th}, w_{cr} \) are selected according to the task and the performance index being monitored, while the gain \( \lambda \) can be experimentally tuned to provide more compliant or more stiff behavior.

In order to find the direction of the force/torque vector \( F_v \) and achieve smooth interaction, it is necessary to calculate the gradient of \( w(q) \) with respect to the Cartesian position and orientation of the end-effector. However, the considered performance indices such as the manipulability index, are calculated from the Jacobian as a function of the joint values \( q \) [4]. An expression of the performance measure as a function of the position and orientation is not available, since the inverse kinematic problem can have multiple or infinite solutions. Instead, a robust algorithm is proposed to approximate the gradient numerically by calculating the vector \( A(q) \) in each of the translational (e.g. in X) and rotational (e.g. \( R_z \)) axes independently:

\[
A(q) = \begin{bmatrix} A_x & A_y & A_z & A_{Rx} & A_{Ry} & A_{Rz} \end{bmatrix}^T
\]
For reasons of clarity, the algorithm for a single Cartesian axis is described, but the procedure is similar for all the principal axes, either translational or rotational. An illustration of the search algorithm for the translational X axis is provided in Fig. 3. When the manipulator is at a joint configuration \( q_0 \in \mathbb{R}^n \), the neighbor joint positions \( q' \) are calculated given an appropriate virtual velocity \( V'_\pm \) in the positive and negative direction of X respectively. Without any loss of generality, the velocity vectors in the X axis are:

\[
V'_\pm = \left[ \pm 1 \ 0 \ 0 \ 0 \ 0 \ 0 \right]^T.
\]

Using the Euler’s integration method, the neighbour joint positions can be found [11], assuming that the end-effector moved with a unit velocity in each of the two directions of X:

\[
q'_\pm = q_0 + J^\dagger V'_\pm \Delta t ,
\tag{3}
\]

where \( J^\dagger \) is the generalized inverse of the \((6 \times n)\) Jacobian matrix \( J(q_0) \) that relates the joint velocities to the translational and rotational velocities of the end-effector. In the case of redundant manipulators \((n > 6)\) the Moore-Penrose pseudo-inverse provides the minimum norm solution for the joint velocities. The integration interval \( \Delta t \) can be set equal to or greater than the sampling period of the control loop.

The new joint configurations \( q'_+, q'_- \) are derived assuming infinitesimal translations of the end-effector by \( \Delta x = V' \Delta t \) in the positive and negative direction of X. For these configurations, the corresponding performance values \( w(q'_\pm) \) are calculated. In order to determine a force \( F'_v \) towards the direction with better performance, the gradient of \( w(q) \) with respect to the X axis of the Cartesian frame of the end-effector is approximated with \( (\nabla w)_{x \pm} \approx \Delta w_{x \pm} / |\Delta x| \).

The value of \( \Delta x \) is a constant that is equal for all axes, therefore, it can be skipped in the following calculations and be compensated in the gain \( \lambda \) of Eq. (2). The value \( \Delta w_{x \pm} \), that is directly proportional to \( (\nabla w)_{x} \), is calculated from the difference between the new performance values and the current performance value \( w(q_0) \):

\[
\Delta w_{x \pm} = w(q'_\pm) - w(q_0) .
\tag{4}
\]

For achieving a smooth virtual force towards the direction with the best performance, \( A_x \) is equal to the maximum value among \( \Delta w_+ \) and \( \Delta w_- \), while the direction of the force is determined from the sign of the \( V'_x \) with the maximum \( \Delta w \):

\[
A_x = \max \{ \Delta w_+, \Delta w_- \} \arg \max \{ \Delta w_+, \Delta w_- \} .
\tag{5}
\]

When the new performance values are smaller than the one of the current configuration \( \Delta w_+ \leq 0, \Delta w_- \leq 0 \) it is an indication that the performance measure is at a local maximum in that axis (as in Fig. 4) and an exception handling mechanism is proposed to prevent virtual forces:

\[
A_x = \begin{cases} 0 & w(q'_+) \leq w(q_0) \ & \& \ w(q'_-) \leq w(q_0) \\ (5), else \end{cases}
\tag{6}
\]

Enabling and disabling the performance constrains with (6) does not present chattering forces, considering that the performance index \( w(q) \) does not present discontinuities in non-singular configurations.

The calculations of Eq. (4)-(6) are repeated for each of the Cartesian axes, for which performance constraints are desired. The main computational cost is consumed for the calculation of \( w(q'_\pm) \), which is measured to be trivial for modern desktop computers using serial CPU execution.

### B. Cartesian Admittance Controller

A typical human-robot cooperation task is conducted by applying forces and torques to the end-effector of the manipulator and by moving it to a desired position and orientation respectively. The virtual forces \( F_v \) can be added to the desired dynamics of the end-effector (Fig. 1):

\[
M_d \ddot{V}_{ref} + C_d V_{ref} = F_h + F_v ,
\tag{7}
\]

where \( V_{ref} \in \mathbb{R}^6 \) is the desired robot Cartesian velocity and \( F_h \in \mathbb{R}^6 \) is the measured external force/torque vector. The gains \( M_d, C_d \in \mathbb{R}^{6 \times 6} \) are constant positive definite diagonal matrices that are selected according to the desired inertia and damping of the end-effector. Variable gains can be used to further improve the operator’s effort [8], [12]. The velocity and force vectors are composed from a linear and an angular part:

\[
V_{ref} = \begin{bmatrix} \dot{p}_{ref} \cr \omega_{ref} \end{bmatrix} \in \mathbb{R}^3, \text{ translational velocities} \quad F_{h,v} = \begin{bmatrix} f_{h,v} \cr \tau_{h,v} \end{bmatrix} \in \mathbb{R}^3, \text{ forces}
\]

\( F_h \) is measured here with a 6-DOF force/torque sensor mounted between the end-effector and the robot. Alternatively it can be estimated from the joint torque sensors.

The desired behavior of Eq. (7) can be implemented using either the impedance or the admittance Cartesian control.
scheme, two controllers with complementary advantages and disadvantages [13]. In this paper, the admittance scheme is selected mainly because it has unacceptable behavior at singular configurations due to the ill-conditioned Jacobian.

In Cartesian admittance control, the calculated operational velocities $V_{\text{ref}}$ are related to the joint velocities of the robot using the inverse differential kinematics. The joint velocities $\dot{q}_{\text{ref}} \in \mathbb{R}^n$ are obtained by

$$q_{\text{ref}} = J^T V_{\text{ref}} ,$$

and by using the Euler integration method, the joint positions are provided to the manipulator’s position control system via incremental joint position commands at the sampling rate $f_s$ of the admittance control loop [11].

The Jacobian of an $n$-DOF, all-revolute joint manipulator consists of:

$$J(q) = \begin{bmatrix} J_T(q) \\ J_R(q) \end{bmatrix} ,$$

where $J_T$, $J_R$ are $(3 \times n)$ matrices representing the parts of the Jacobian that map the joint velocities to the translational and rotational velocities of the end-effector respectively. These sub-matrices have different physical units and as a result, their application in a single performance index can be problematic. Therefore, the translation and the rotation part, should be represented by separate indices.

1) Cooperation with translation and indifferent orientation: Considering the kinematic manipulability as the performance index [4], two separate indices can be formed:

$$w_{T_{\text{ref}}}(q) = \sqrt{|J_T J_T^T|} , \quad w_{R_{\text{ref}}}(q) = \sqrt{|J_R J_R^T|} ,$$

where $w_{T_{\text{ref}}}, w_{R_{\text{ref}}}$ are the translational and rotational manipulability measures in the weak sense [14]. These indices quantify the end-effector’s ability to translate and rotate, but without considering its orientation and its position respectively. Specifically, the translational manipulability in the weak sense can be used in human-robot cooperation tasks, where only the externally applied forces by the operator are considered ($F_h = f_h \in \mathbb{R}^3$) and the end-effector’s orientation is indifferent. In that scenario, the kinematical manipulability of the Cartesian admittance controller of Eq. (8) become:

$$\dot{q}_{\text{ref}} = J_T^T \dot{p}_{\text{ref}}$$

and the rotational manipulability is meaningless to be calculated since no constraint torques should be applied.

2) Cooperation with translation and constant orientation: A cooperation task that requires constant orientation of the end-effector during translational movements, adds a further restriction that is represented from the null space projection $(I - J_R^T J_R)$. Similarly, only the operator’s forces are considered, but the differential kinematics mapping is represented from the augmented translational Jacobian:

$$\tilde{J}_T = J_T (I - J_R^T J_R) ,$$

and Eq. (8) becomes:

$$\dot{q}_{\text{ref}} = \tilde{J}_T^T \dot{p}_{\text{ref}} .$$

The appropriate manipulability measure for the end-effector’s translation that includes the additional restriction (translational manipulability in the strong sense [14]), which is also used in the experimental evaluation, is calculated from the augmented Jacobian:

$$w_{T_{\text{ref}}}(q) = \sqrt{|J_T (I - J_R^T J_R) J_T^T|} . \quad (14)$$

III. EXPERIMENTAL EVALUATION

To validate the proposed method for performance constraints, two experiments are conducted using a KUKA LWR IV 7-DOF manipulator in cooperation with a human. The first task involves the guidance of the end-effector with a translation towards a singular configuration (Fig. 5a). The second one includes guidance of the end-effector to follow an elliptic path (Fig. 5b). These tasks have been positioned accordingly so that the robot kinematics limit the robot’s performance on certain configurations. The tasks are performed under Cartesian admittance control, maintaining constant orientation using Eq. (13), initially without any constraint forces. Then, they are repeated with the proposed performance constraints enabled, using the translational manipulability index $w_{T_{\text{ref}}}(q)$ of Eq. (14).

The manipulability index being equal to zero, is a necessary and sufficient condition for a singularity [3]. Therefore, the critical manipulability value is selected equal to $w_{\text{cr}} = 0.01$ to guarantee that the robot never reaches a singularity and that the Jacobian matrix has full rank for calculating the inverse differential kinematics. The threshold is set slightly higher, to $w_{\text{th}} = 0.025$ and the gain is experimentally determined to provide smooth constraining forces for $\lambda = 100$. The overall admittance control loop with the proposed constraints operates at $f_s=1$ kHz, with a processor i5-3570@3.4 Ghz, on Linux OS with real-time kernel. A simulation of the proposed algorithm that uses the manipulability index to prevent the robot from reaching low performance configurations is provided as supplementary material in [15]. A demonstration of the experiments is available in the accompanied video [16].

A. Translational Linear Movement

In the first experimental scenario a human grasps the handle and applies a forces $F_h$, along the X axis of the Cartesian frame attached to the end-effector, to guide the robot towards the boundaries of its workspace (Fig. 5a). There, the robot is at a boundary singularity because two of its joint axes coincide. This is a very common problem that is encountered in human-robot cooperation, particularly with operators who are unfamiliar with kinematic singularities.

Initially, no constraints are implemented and when the robot reaches the singular configuration the commanded joint velocities $\dot{q}$ become too high, exceeding the permissible value (Fig. 5c). The robot automatically engages the emergency joint brakes as a reaction of the high joint velocities and the task is interrupted. This undesirable situation is corrected when the proposed performance constraints are enabled (Fig. 5c). The constraints produce smooth forces $F_v$. 

3052
that prevent the operator from approaching a configuration with critical manipulability (Tab. I). The joint velocities no longer exceed the permissible values and the maximum measured velocity (observed in joint 4) is reduced by 76%. The constraints can be decomposed into a major force along the X axis and a smaller one along the vertical axis Z. This occurs because the gradient of the manipulability index produces a force along the Z+ direction that can also improve the manipulability. No virtual force is applied along the Y axis, where the index is at a local maximum.

B. Elliptic path following

The second experiment involves cooperation with the robot for following an elliptic path on a horizontal plane. Part of the task is placed close to a low manipulability configuration in order to investigate any performance improvements with the virtual constraints. During the task only translational motion in the XY plane is allowed with constant orientation.

It can be observed that even though the manipulator does not go through a boundary singularity in the experiment without the constraints, the joint velocities as well as the joint accelerations are quite high because of the low manipulability index at certain configurations. The results of low manipulability in the cooperation appear as involuntary oscillations of the end-effector that deteriorate the cooperation. When the performance constraints are enabled, the maximum joint velocities are reduced by 44% (Tab. I) because the virtual

---

**(c)**

Fig. 5: Moving the end-effector (a) with a translation towards its workspace boundaries and (b) in an elliptic path, with and without the proposed method for performance constraints. The corresponding graphs (c) and (d) illustrate the joint velocities $\dot{q}$, the operator’s forces $F_h$, the constraining forces $F_v$ that are applied from the robot to the operator, and the relative position of the end-effector that is colored with the instantaneous translational manipulability index. The scale of the vertical axes between the graphs of the experiments with and without constraints is the same for comparison purposes.
TABLE I: Comparison of the joint velocities, manipulability and operator’s forces with and without the performance constraints.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>No constraints</th>
<th>With constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum $\dot{q}$ (rad/s)</td>
<td>2.0</td>
<td>0.84 0.48 0.47</td>
</tr>
<tr>
<td>Min manipulability</td>
<td>0.003</td>
<td>0.006 0.021 0.020</td>
</tr>
<tr>
<td>Maximum $F_h$ (N)</td>
<td>4.24</td>
<td>4.71 11.16 13.94</td>
</tr>
</tbody>
</table>

forces limit the workspace above the critical threshold.

C. Discussion of Results

It is worth noting that in both tasks with enabled performance constraints, the maximum force applied by the operator is significantly increased compared to the experiments without the constraints (Tab. I). When the manipulability drops below the threshold, the operator gradually increases the effort in order to overcome the constraining forces. The asymptotic increase in the constraining forces, which is also perceived as increase in the apparent stiffness of the end-effector (from Eq. (2)), indicates to the operator that the end-effector cannot be moved further, towards directions with lower performance. Eventually, the operator alters the path in order to complete the task, but has become aware of the robot performance limitations via the force feedback from the constraints.

Although the proposed method is demonstrated with a redundant manipulator in boundary singularities, the same algorithm can also encounter internal singularities that occur inside the workspace of a serial manipulator using an appropriate performance measure. The introduced performance constraints provide haptic feedback to the operator to avoid singular configurations and facilitate comfortable cooperation. The observations from the experiments suggest that an unskilled operator without knowledge of robot science and kinematic singularities can acquire an intuitive knowledge of the robot performance simply by interacting with it. Then, the operator can reallocate the task to areas of high performance for improved cooperation.

The proposed method calculates the constraining forces without taking into consideration subjective parameters such as the operator’s velocities or external forces that also include measurement noise. Since the method only relies on a manipulator’s performance measure and produces asymptotic increase of the apparent stiffness of the end-effector in the directions of lower performance, it is guaranteed that the operator cannot guide the robot below the user-defined critical performance value. The experimental tasks presented here were conducted using one subject, as a proof of concept for the proposed performance constraints. Nevertheless, the promising results encourage us for conducting a thorough study with multiple subjects, in order to investigate and compare the perception of the constraining forces by skilled and unskilled operators. Some initial experiments with very high Cartesian velocities, avoidance of inner singularities and varying orientation of the end-effector demonstrated successful performance.

IV. Conclusion

Motivated by the need to provide feedback to the operator about the performance of the manipulator during cooperation and particularly for the prevention of kinematic singularities, a method is proposed to impose manipulator performance constraints in Cartesian admittance control. This method addresses an important issue for the advancement of human-robot interaction towards safe, efficient and comfortable cooperation.

A robust algorithm is presented that calculates the virtual constraining forces online, by approximating the gradient of the performance measure with respect to the Cartesian frame attached to the end-effector, and without the need of an analytic expression of the index (which is not always available). The overall scheme is validated using a redundant 7-DOF robot under Cartesian admittance control in two cooperation tasks with constant orientation, using the manipulability index for singularity avoidance. The method can also be applied in n-DOF serial manipulators using Cartesian admittance or impedance control for translation and/or rotation of the end-effector.

REFERENCES